## 6. Solution of the Equations

Solution of the equations is a simple matter under the present physical assumptions. One begins by deriving an equation for the radius of curvature from the fluid mechanics of Sections 3 and 4. The friction law (14) can be used to eliminate stress from the momentum equation (10), with the result that

$$\frac{1}{\mu_w} \frac{d}{d\theta} R(p_s - p_a) = R(p_s - p_a) + R(p_a - p_v) \quad .$$

Both $p_a$ and $p_v$ are negligible for rock-cutting applications, so the second term on the right-hand side can be dropped. The rest of the equation is immediately integrable:

$$R(p_s - p_a) = R_o (p_s - p_a)_o \, e^{\mu_w(\theta - \theta_o)} \quad , \qquad (26)$$

where the subscript o refers to conditions at the beginning of the cut. From equation (8),

$$R_o (p_s - p_a)_o = \rho u_o^2 d_o \quad ,$$

since the jet is initially uniform and has a speed $u_o$. Then from the Bernoulli equation (6),

$$R_o (p_s - p_a)_o = 2(P_o - p_a) d_o \quad ,$$

and equation (26) becomes

$$R(p_s - p_a) = 2(P_o - p_a) d_o \, e^{\mu_w(\theta - \theta_o)} \quad .$$

A second application of (14) eliminates $(p_s - p_a)$ in favor of $\tau$:

$$(p_s - p_a) = \tau/\mu_w - (p_a - p_v) \quad .$$

Thus

$$R = 2 \mu_w \frac{d_o P_o}{\tau} e^{\mu_w(\theta - \theta_o)} \quad , \qquad (27)$$

where $p_a$ has been neglected compared with $P_o$ and $(p_a - p_v)$ compared with $\tau/\mu_w$, in conformity with the approximation that led to (26). Equation (27) is the final result of the fluid-mechanical arguments.